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4 K and J-Integral Approach for Fatigue-Creep Crack Growth Studies

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Abstract

Fracture mechanics principles are being applied to understand the failure behavior of components subjected to fatigue loading at elevated temperature. Linear elastic and elastic-plastic parameters such as K and J-integral are normally employed. At low temperature and higher frequency of loading, stress intensity factor K can characterize the material behavior. At higher temperature and low frequencies, time dependent effects will come into play. Crack growth models have been developed to characterize the material behavior incorporating linear and nonlinear parameters. No clear cut limits for the application of different parameters are established. There is lack of experimental data for the whole range of temperature at which the components are supposed to function.

Keywords: Fracture Mechanics, Fatigue Analysis, Material Behavior, Cracks.

الملخص

الهدف من هذا البحث تطبيق مبادى ميكانيكا التكسير Fracture Mechanics Principles لفهم إخفاق سلوك Failure Behavior المواد والعناصر المرتبطة بوهن الأحمال الديناميكية Fatigue Loading عند حرارة مرتفعة Elevated Temperature . المعاملات الخطية المرنة Linear Elastic parameters والمعاملات المرنة اللدنة Elastic- Plastic Parameters عادة تستخدم عند حرارة منخفضة وتردد عالي للحمل الديناميكي، عامل شدة الإجهاد Stress Intensity Factor يستطيع أن يصف سلوك المادة Material Behavior. عند حرارة عالية وترددات منخفضه، تأثيرات الزمن المعتمد Time Dependent effects سوف توخذ بالحسبان. موديلات نمو التصدع Crack Growth Models طورت لتصف سلوك المادة مجسدة المعاملات الخطية واللاخطيه Linear and Nonlinear Parameters. لا يوجد هناك وضوح محدد و قاطع لتطبيق المعاملات المختلفة. هناك قصور في وجود القيم الإختبارية تشمل المدى الكامل لدرجات الحرارة و التي يجب إن تصف أداء القطعة.

أن انتشار التصدع والتكسر في المواد وأسبابها حللت وقدم نقاش متكامل وشامل لربط المواضيع مع بعضها وتقديم خلاصة بشكل مترابط ومتناسق.

Nomenclature

A -Constant of crack growth law,
A -Crack length,
B -Specimen thickness,
B -Constant of crack growth law,
C -Material constant of pairs law,
C^* -Modified J integral,
da/dN -Crack growth per cycle,
f(a/w) -A function depending on the configuration of the specimen,
J -J integral,
K -Stress intensity factor,
K_{max} -Maximum value of SIF,
K_{open} -value of SIF at which the crack is just fully open,
k -Exponent in the secondary creep hardening law,

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m -Paris law constant,
N -Number of cycles,
p - a constant,
R -Stress ratio,
r_y -Plastic zone size,
t -Time,
t_1 -Transition time,
t_h -Hold time,
Δ -Deflection,
ΔJ_c -Cyclic J for creep,
ΔJ_f -Cyclic J for fatigue,
Δk -Stress intensity factor range,
ΔJ -Cyclic J integral,
σ -Stress,
C -a constant.

Abbreviations: LEFM – linear elastic fracture mechanics, **SIF –** Stress intensity factor.

1. INTRODUCTION

In the present day fracture mechanics principles are applied to understand the fatigue behavior of material, subjected to different types of loading and environments. Fatigue behavior is characterized in terms of fatigue crack growth parameters such as ΔK, ΔJ and C^*. When the temperature is lower than homologous temperature, the effect due to creep is very much small, the crack tip stress field is dominantly elastic and fatigue behavior can be studied using ΔK. But at elevated temperature as the time passes the uncracked ligament will become plastic. Elastic plastic parameter such ΔJ has to be used for characterizing crack growth. To characterize the fatigue behavior for the whole range of operation of components, equations have been developed combining different fracture mechanics parameter. In this paper we present different fracture mechanics methodologies available for characterizing fatigue creep behavior of metallic material.

2. ΔK APPROACH

A thorough knowledge of fatigue crack growth at room temperature is a prerequisite to the study of high temperature crack growth. At low temperature same mechanisms operate. Range of SIF ΔK is being used to characterize fatigue crack growth behavior at room temperature from a long time Crack growth law similar to Paris equation has been developed for various material using different specimen geometries. Paris law is given by.

$$da/dN = C(\Delta K)^{m} \quad (1)$$

$$\Delta K = \Delta \sigma \sqrt{a} \; f(a/w) \quad (2)$$

Plot of Log da/dN versus log ΔK is a Sigmoidal curve showing three stages of crack growth (Fig.1). First stage is associated with Threshold effects. Second stage represents stable crack growth. Third stage represents rapid unstable crack growth. Most of **LEFM** concepts are applied to stable crack growth region. Here da/dN is linear, independent of frequency and thickness. Crack growth law are, equations of linear fit to stable crack growth region. The Sigmoidal curve is also called Fatigue Crack Growth Curve **(FCGR)**. The extent of plastic deformation near the crack tip governs the applicability of different Fracture Mechanics parameters. For fatigue at room temperature plastic zone size is given by,

$$r = 1/8\pi \left[\left(k_1 / \sigma_y \right)^2 \right] \quad (3)$$

this is a very small region surrounded by a dominant elastic field. Range of crack tip opening displacement **(CTOD)c** is also used to characterize fatigue crack growth. It is given by,

$$\text{(for } R = 0) \quad (CTOD)_c = 0.22\, \Delta K^2/(\sigma_y E) \tag{4}$$

Understanding the effect of stress ratio R, frequency of loading and waveform of loading on the fatigue crack growth is very important. For a given ΔK more positive the stress ratio R the higher the crack growth rate. Lower frequency of loading gives higher crack growth rate. Loading wave from in which portion of tensile load per cycle is longer, gives higher crack growth. Positive saw tooth, triangular, trapezoidal wave forms are the examples. Crack closure that takes place due to crack tip plasticity explain the effect of R ratio, frequency of loading and wave form of loading.

Due to plastic deformation, crack tip remain closed until tensile load reaches a certain level. The level of stress at which the crack tip is just fully open is denoted by $\boldsymbol{\sigma_{op}}$. During unloading portion of the cycle the crack tip close even before the load is zero. Taking these things into considerations range of effective stress intensity factor has been defined as.

$$\Delta K_{eff} = K_{max} - K_{open} \tag{5}$$

When stress ratio is low (but > 0) there would not be much environmental effect. The crack tip remain closed for relatively longer period, during tensile portion of a cycle. At higher frequency of loading there will not be much time for environmental interactions of the crack tip.

At higher value of R and at low frequency, the period of crack tip remaining open per cycle increases. There will be enough room for environmental interaction such as oxide growth etc. This increases crack growth rate. In case of positive saw tooth, triangular and trapezoidal wave forms the period of crack tip remaining open is higher and enhanced due to effects of crack closure. This gives rise to higher crack growth. In tension-going part of a fatigue cycle, intense plastic flow occurs in the maximum shear direction. At peak strain decohesion occurs. When stress reversal takes place these gives rise to striations. Formation of striation is accepted to be the way of crack propagation in the stable crack growth region. This mechanism is known to operate during elastic plastic fatigue as local compression occurs due to crack closure. Striations are a manifestation of localized crack tip plastic deformation.

How the temperature affects these mechanisms is the question. At higher temperature yield strength decreases giving rise to increased plastic zone size. Due to this crack closure effect is increased giving room for increased environmental interaction. At higher positive R values the effect of compression of plastically stretched material may create severe loading condition of the crack tip. Crack growth by striation formation will be more pronounced. Dominating elastic field surrounding the plastic zone may change to a wake of plastically deformed material. When frequency of loading is low and there is interspersed hold period at tensile maximum, elastic situation breaks down and time dependent creep effects will govern the crack growth. ΔK has been derived assuming σ_y to remain constant at a given temperature and the stress field remain dominantly elastic. As seen in the above discussion these conditions are not true at elevated temperature. ΔK approach will be invalid for characterizing crack growth at elevated temperature.

At low temperatures and higher frequency, the effect of environmental interaction will be small. Change in yield strength will not be much, ΔK can characterize the fatigue crack growth. But clear cut upper limit of temperature and lower limit of frequency for the applicability of ΔK is not yet established. There is lack of information and experimental data.

The effect of increasing temperature upon fatigue crack propagation rate is generally to cause an increase at given ΔK. The extent depending upon the material. Many investigators [Saxena et al., 1981; and Swaminathan et al., 1982] have developed models using ΔK to predict the influence of hold time on fatigue crack growth behavior at elevated temperature.

$$(da/dN)_h = C(\Delta K)^m + A\,(\Delta K)^{2p}\,(t_h)^{1-p} \quad (6)$$

C, M, A, P etc. are constants which can be experimentally determined. For A 470 class 8 steel at 539 °C for 5 sec rise and decay time,

$$(da/dN) = 1.49 \times 10^{-7}\ (\Delta K)^{2.35} + 1.18 \times 10^{-6}\,(\Delta K)^{1.27}\, t_h^{0.365} \quad (7)$$

These equations are valid up to 60 secs hold time. For longer hold time the model is extended incorporating the transition time t_1 and the energy rate line integral C*. Transition time is the time required for small scale plastic stress strain field near the crack tip to trans forming into large scale plasticity. It is derived by Reidel and Rice [Riedel, 1980].

$$t_1 = [\ k^2\ (\ 1-e^2)]/\ E\ (\ 1+k\ )\ C^*] \quad (8)$$

$$(da/dN)_h = C\ (\ \Delta K\ )^m + A(\Delta K)^{2p}\, t_1^{(1-p)} + C\ (C^*)^a\ (\ t_h - t_1\ ) \quad (9)$$

Many experimental investigations [Shahinian, 1976; and Michel, 1976] indicate ΔK approach to be valid at low temperature. Fig. 2 shows plot of da/dN versus ΔK for Inconel 718 at 649 °C for various hold times and at several loads.

3. J-INTEGRAL APPROACH

At temperatures, near and above 0.5 times melting temperatures plasticity effects at the crack tip become more appreciable. ΔK cannot provide reliable prediction of component life. Cyclic J-integral, which is an extension of J integral defined for static fracture has been applied to characterize fatigue creep behavior [Dowling, 1976]. Cyclic J is denoted by ΔJ. The irreversible deformation occurring during unloading portion of cyclic loading makes application of J-integral theoretically invalid, since the integral will no longer be path independent. In general if one assume that damage occurring during unloading part of the cycle is reflected in loading part of the cycle.

Moreover the plastic zone attains a stable condition as represented by saturated hysteresis loop due to cyclic loading and there is no history dependent behavior. Each new hysteresis loop can be considered independently of previous deformation history. For conditions of wide spread plasticity ΔJ represents the elastic plastic work required to deflect the specimen and is a measure of the intensity of the characteristic strain filed. Area under the rising portion of P-Δ hysteresis loop (A_t) reflects all the changes occurring at the crack tip and is used in determining the value of ΔJ.

$$\Delta J = (2A_t/B_t) f(a/w) \tag{10}$$

Figure 3 shows different ways of determining ΔJ after N cycles. Many investigators [Shao lun and Ji-Zho; El Haddadand and Mukherjee, 1983; Tanaka and Nakata, 1983; Kaisand and Mowbray, 1979] have used ΔJ for characterizing crack growth under conditions of excessive plastic deformation. Plot of da/dN versus ΔJ shows only two stages of crack growth as against three stages of da/dN versus ΔK. The use of ΔJ significantly reduces scatter in data. The usual stage III behavior maps into a straight line continuing stage II into the region where **LEFM** is not normally valid. In some cases J_{max} gave better correlation. Attempts are being made to use rate dependent J integral C* for characterizing time dependent crack growth. Few investigators [Ryuichi and Takayuki, 1988] have adopted strain range partitioning method and split ΔJ into fatigue J integral ΔJ_f and cyclic creep J-integral ΔJ_c (Fig.4). For long crack, fatigue-creep interaction was found to be less important.

Some theoretical analysis are also available in support of cyclic J-integral. Finite element elasto-plastic analysis techniques are usually adopted, apart from other integration techniques [German et al., 1985].
In theoretical investigations the effect of crack closure is not given due consideration. In experimental methods it is accounted for by considering the area of P-Δ plot above inflection point at which crack starts opening while loading and closes while unloading.

4. CONCLUSIONS

1. Up to certain temperatures stress intensity factor K can be used to characterize fatigue crack growth behavior at elevated temperatures.
2. Since plasticity effects are bound to be present at the crack tip at elevated temperatures J – integral or ΔJ integral can be used instead of K. these parameters can characterize crack growth behavior at lower temperatures as well as at higher temperatures. They can characterize both time independent and time dependent crack growth. Very little investigation has been done in this area.
3. in the present time modified J – integral or C^* integral has been applied to studied state creep crack growth under static load.
4. Some authors have proposed some models in terms of Δk and C^* . they express da/dN as a function of Δk, t_1 and C^* . t_1 is the time of transition. Up to time t_1 ΔK will be the controlling parameter, after that C^* will be the parameter. Any how enough experimental work has not been done in verifying such models. Approaches of this type are fewer in number.
5. Very few attempts are made to describe the crack growth behavior for the entire range of temperatures in which the material is supposed to operate. No clear cut limits are set to the application of K, $K_{max.,}$ ΔK, J, $J_{max,}$ ΔJ, for high temperature application.

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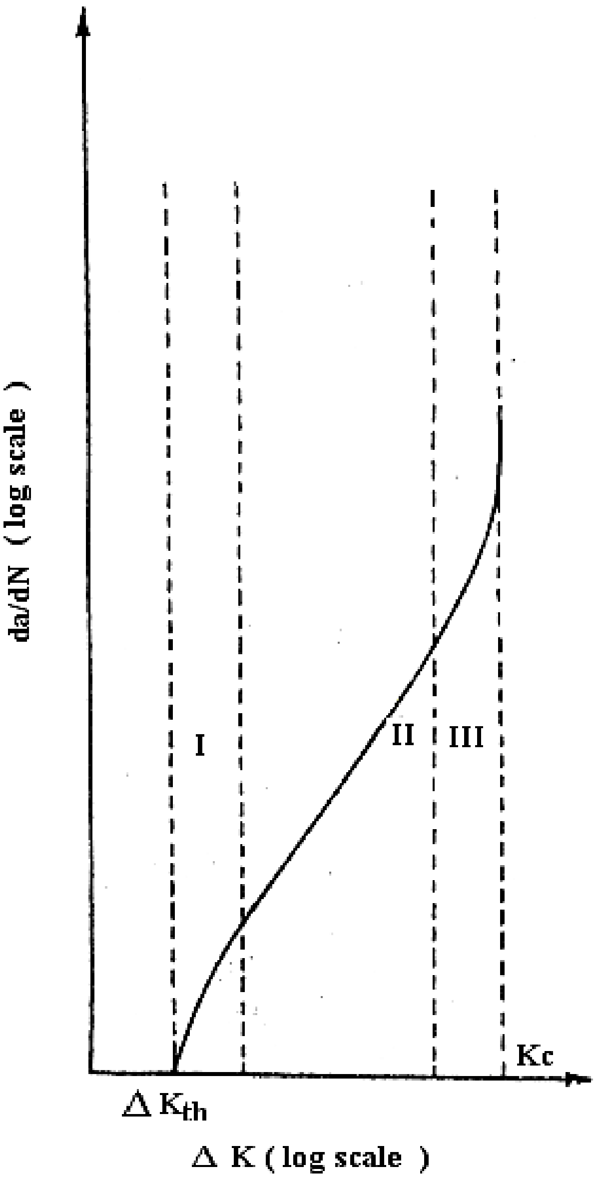


Fig. 1: Plot of da/dn vs DK for room temperature, showing three regions of crack growth [Julie et al., 1990]

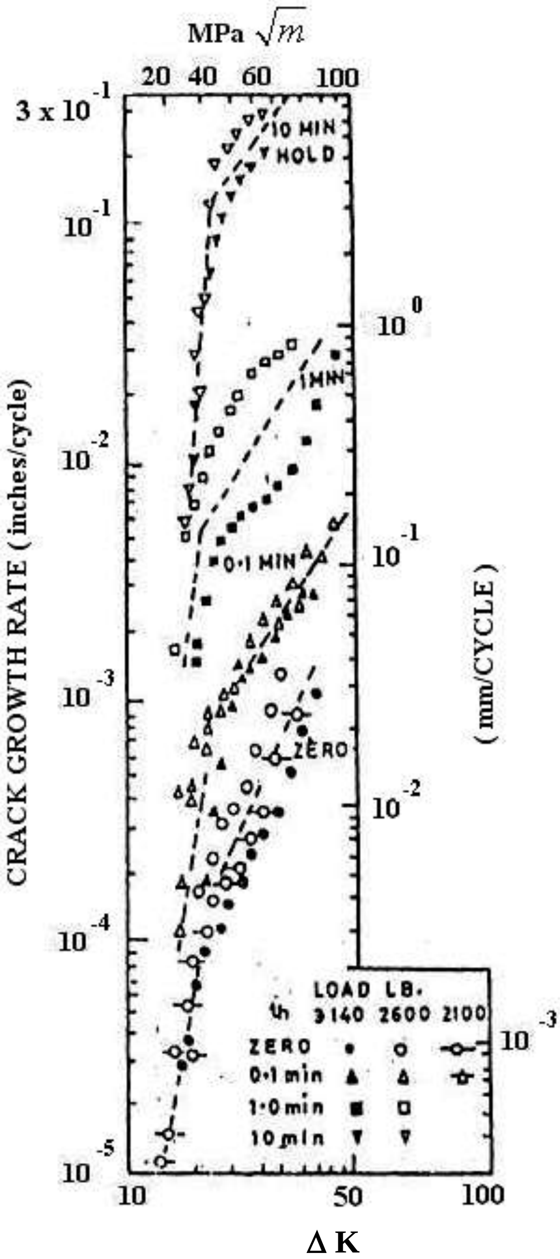
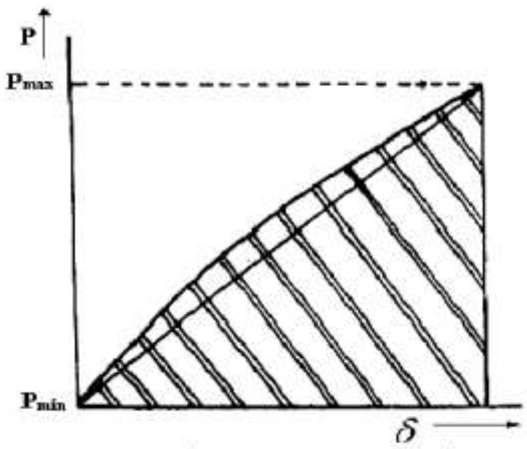
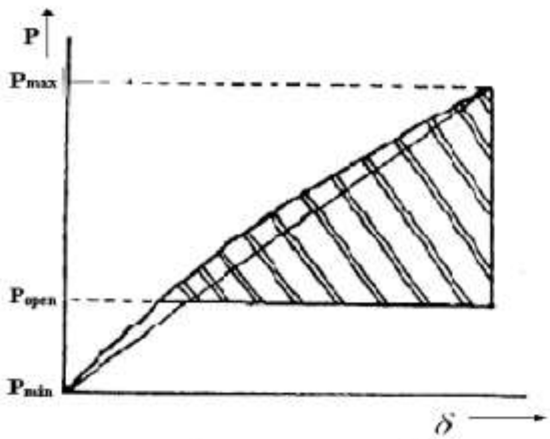


Fig.2: Effect of hold time on crack growth rates (da/dN) as a function of ΔK for several loads at 649°C (1200°F) [Shahinian and Sadanada, 1976]



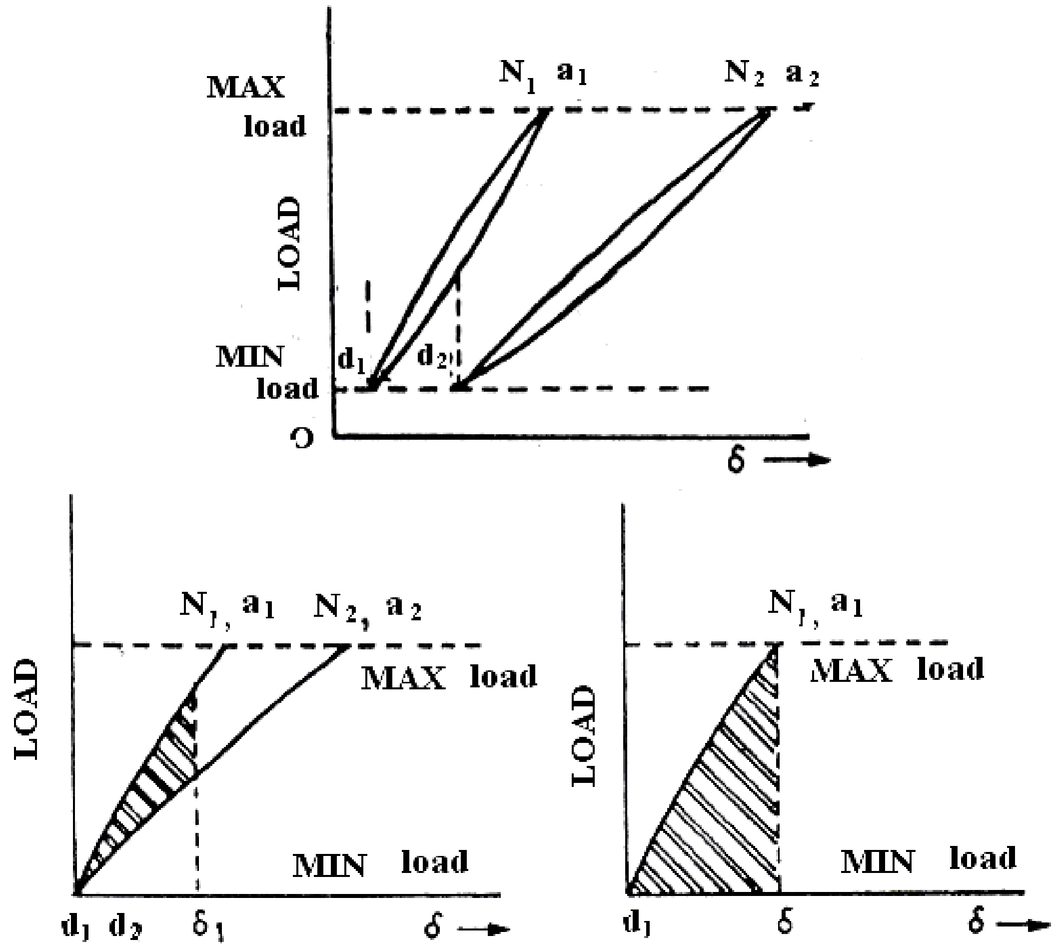
$$\Delta J = 2\left(\frac{HATCHEDAREA}{B_b}\right)$$

(a): Rice's definition rack of Δ J



$$\Delta J_{eff} = 2\left(\frac{HATCHEDAREA}{B_b}\right)$$

(b): Δ J considering rack closure



(c) Δ J for N cycles [7]
Fig. 3

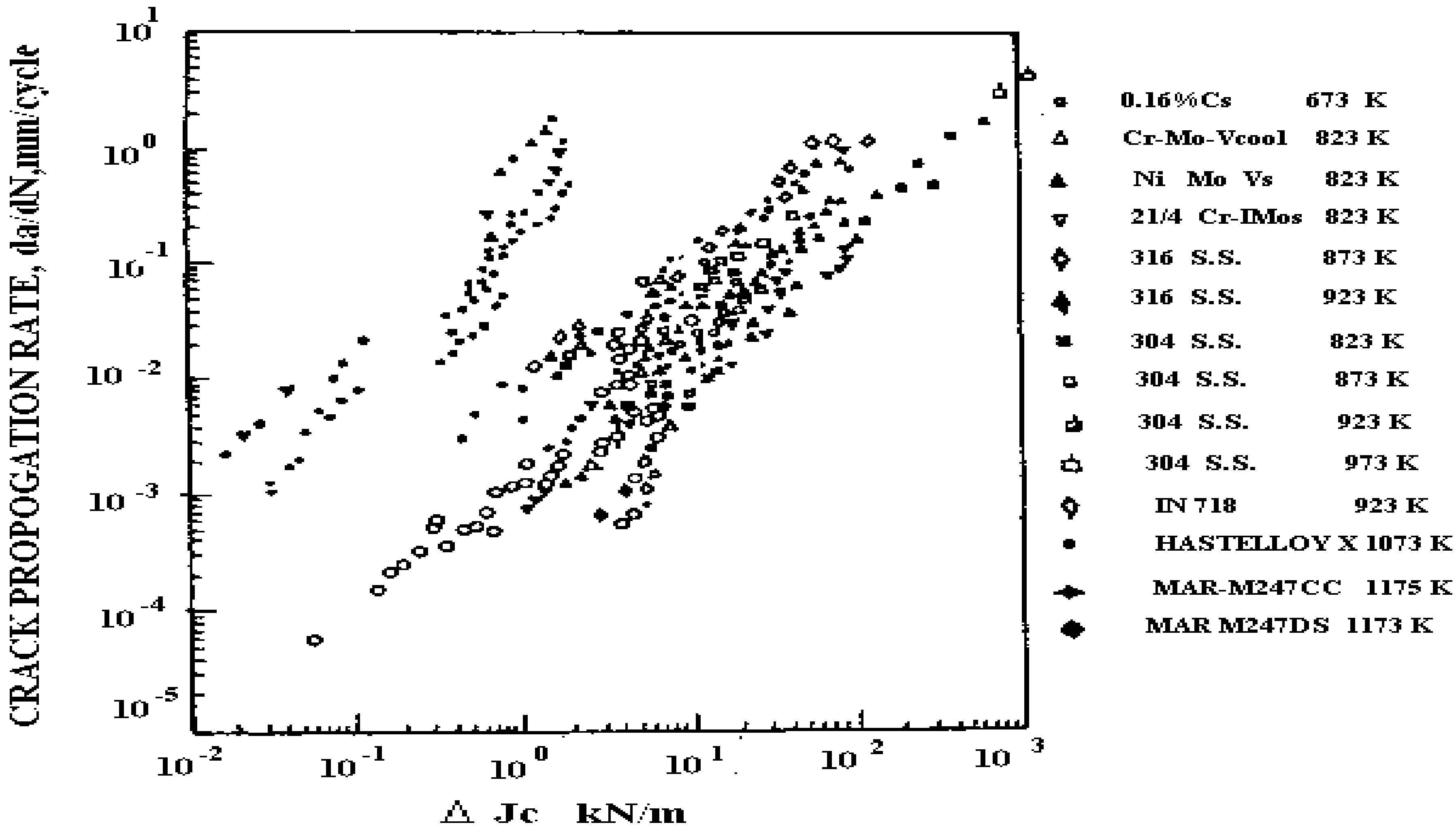


Fig. 4a: Relationship between da/Dn and ΔJ_C in creep with a cyclic stress change [Ryuichi and Takayuki, 1988]

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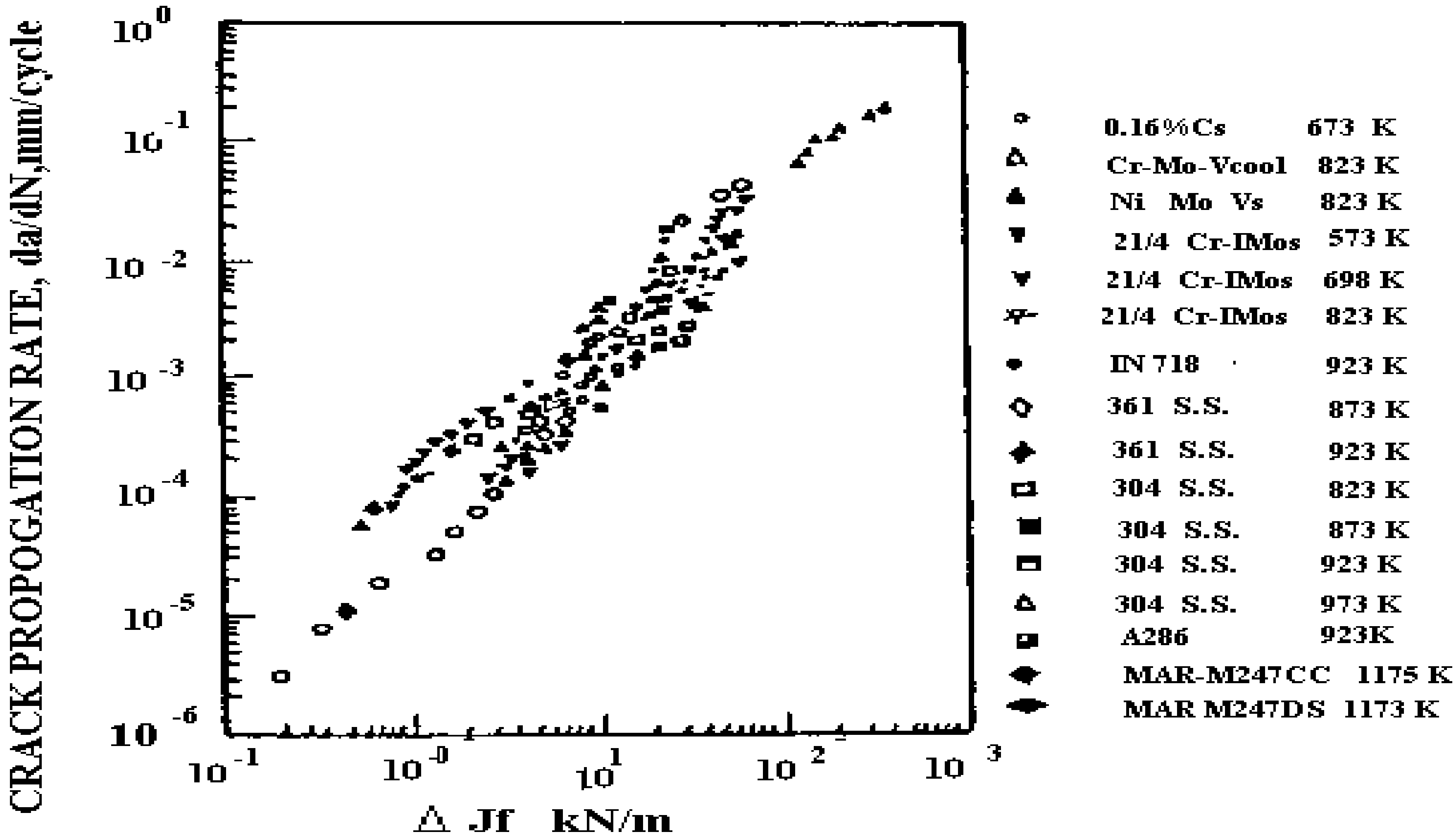

Fig. 4b: Relationship between da/Dn and ΔJ_f in the fatigue of metallic materials [Ryuichi and Takayuki, 1988]