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2 A Study of Parameters of Optimized Helical Gears

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Abstract

This paper is an extension of earlier work of A. Kader [2003], which dealt with the non-linear optimization technique for design of a compact helical gear set. The concept of design space based on module was developed. In the present work, the effects of different input parameters such as power transmission, gear speed, gear ratios, pressure angle, helix angle, face width, and material properties on the optimal volume have been studied. It is observed that the optimal volume is directly proportional to the power transmission and inversely proportional to the gear speed, the pressure angle and helix angle. Also different materials properties are used to study its effect on the optimal values of the volume and the corresponding helix angle, face range, number of teeth on the pinion, module size, and face width for both the pressure angles 20º and 25º have been evaluated. An attempt has also been made to correlate the modes of failure and material properties in different conditions.

Keywords:

الملخص

هذا البحث تمديد للعمل السابق (عبد القادر 2003م), والذي يستعرض عن تقنيّة الحل الأمثل للمسألة اللاخطيّة و ذلك للحصول على المسننات المائلة ذات الحجم الأقل باستخدام مفهوم الحيز الفراغي للتصميم الأمثل وفي هذا البحث أجريت دراسة شاملة للتأثيرات الناتجة بسبب المعطيات المختلفة مثل قوة العزم سرعة المسننات نسب سرعة المسننات و كذلك أذخال عوامل مختلفة مثل زاوية الضغط زاوية ميلان السنّة عرض السنّة و أيضا درست تأثيرات خواص المعادن المختلفة على الحجم الأمثل للمسنن المائل

لوحظ في الدراسة أن حجم المسنن الأمثل يتناسب مع قوة العزم تناسبا طرديّا ويتناسب مع سرعة المسنن تناسبا عكسيا مع أخذ كل اعتبارات الفشل عند التصميم الأمثل للمسننات و في الأخير فرّزت الدراسة المعادن المختلفة من حيث أعطائها أقل حجما للمسننات مراعاة العوامل المختلفة المشار أليها أعلى

Nomenclature

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C_v	Factor for scoring mode of failure
I	Durability geometry factor
J	Bending geometry factor
K_v	Factor for bending mode of failure
m	Module, mm
m_n	Normal module of helical gear, mm
N	Number of teeth
n	Speed, rpm
T	Torque transmitted, kW
V_p	Optimal volume of the pinion, cm^3
ϕ	pressure angle
ϕ_t	Transverse pressure angle
ψ	Helix angle in degree
λ	Length to diameter ratio
σ_b	Bending stress, N/mm^2
σ_c	Surface compressive stress, N/mm^2
b	second subscript for bending design
s	second subscript for scoring
1	Subscript for Pinion.

1. INTRODUCTION

In general the most desirable gear set is the smallest one that will perform the required task. Smaller gears are easy to make, run more smoothly due to smaller inertial loads and pitch line velocities, and are less expensive. Therefore, a desirable design objective has always been to minimize the size (volume) of the gear.

[A. Kader, et. al., 2003] developed a software to design and optimise gear set. Design space in terms of module as a main design parameter to minimize the volume for helical gear set with pressure angles of 20° and 25°, for full depth form was discussed. Present work is based on the work of [Jog and Pande, 1989], who gave a design strategy for the computer aided design of compact helical gear sets. Using optimization techniques, fundamental gear design parameters such as number of teeth on the pinion, helix angle and diametric pitch were selected and subjected to constraints on bending stresses, contact stresses and involutes interference limits.

[Nigam, et al., 1992] presented a graphical optimization technique used for designing compact spur gear set by developing a non-linear programming. Centre distance was taken as the objective function of the problem subjected to kinematics constraints of bending stress, scoring wear, pitting stress and involutes interference.

[Savage, Coy and Townsand, 1982] Carried a literature review and presented a well-defined method based of optimization considerations. A design space for spur gear was defined in terms of the number of teeth on the pinion and the diametric pitch. This space was then combined with the objective function of minimum centre distance to obtain an optimal design region. This region defines the number of pinion teeth for the most compact design.

[Carroll and Johnson, 1984] expanded the Savage, Coy and Townsand model, to include the AGMA geometry and dynamic factors. The method developed was applicable to the tooth systems in which the addenda and the dedenda are inversely proportional to the diametric pitch. The design objective was to minimize pinion diameter subjected to the conditions of: (a) no involutes interference, (b) contact stress does not exceed certain limits, and (c) gear does not fail in bending fatigue.

[Eggeman, 1984] developed three interrelated mathematical models to determine the size and shape of a helical gear tooth. These models where combined into a design procedure to produce a compact high load capacity helical gear set by using an optimization technique.

[Savage, Coy, Kinnel and Lattirne, 1994], presented an important paper that proposed an optimal design of compact spur gear reduction that included the selection of bearing and shaft proportions in addition to the gear mesh parameters. Designs for single mesh spur gear reductions where based on optimization of system life, system volume and system including gear, support shafts and bearings.

[A. Kader, et al.,1998], critically studied the behaviour of mode of failure for large range of parameters variations for 20 commonly used gear materials for both 20^o and 25^o pressure angle, full depth spur gears under optimised conditions. Minimum centre distance has been chosen as the optimisation objective function and an in-depth study of bending, pitting and scoring mode of failures has been attempted.

This work is an extension of the work carried by the author [A. Kader, et. al., 2003], in which the software developed earlier is used to study the effect of different gear design parameters on the optimised conditions and to have a better insight into the problem, for the optimum condition of any required helical gear design set. This helps the designers to select the required gear material by adjusting the different design parameters to get optimal results.

2. MATHEMATICAL MODELLING OF THE DESIGN SPACE EQUATION AND ITS GENERATION

For the sake of continuity, the objective function and the design constraint equations, which were used in the generation of design space as given by [A. Kader et. al., 2003] are reproduced below:

The design objective function in terms of volume is,

$$V_p = \frac{\pi^2 m_n^3 N_1^2 \lambda^3}{4 \cos^2\psi \sin\psi} \quad (1)$$

thus the design objective is to find out the optimal values of number of teeth on the pinion and the module, whose values when substituted in the above objective function give an optimal minimum volume for the given set of input parameters and constraint conditions.

For the design constraints the bending limit, a lower bound on the module, can be determined by ,

$$m_{n_b} \geq \left[\frac{2T \cos^2\psi \sin\psi}{\sigma_b J \lambda N_1^2 K_v} \right]^{1/3} \quad (2)$$

or in terms of power

$$m_{n_b} \geq \left[\frac{P \cos^2\psi \sin\psi}{\pi n \sigma_b J \lambda N_1 K_v} \right]^{1/3} \quad (3)$$

The constraint for the contact stress for the lower limit on the module in terms of Hertz stress is given as,

$$m_{n_p} \geq \left[\frac{2T C_p^2 \cos^2\psi \sin\psi}{\pi \sigma_c^2 I \lambda N_1^2 C_v} \right]^{1/3} \quad (4)$$

or in terms of power as

$$m_{n_p} \geq \left[\frac{P C_p^2 \cos^2\psi \sin\psi}{\pi^2 n \sigma_c^2 I \lambda N_1 C_v} \right]^{1/3} \quad (5)$$

and the minimum number of teeth required to prevent involute interference is constrained by,

$$N_1 \geq \left[\frac{2\cos\psi}{\sin^2 \varphi_t} \right] \qquad (6)$$

To generate a two dimensional design space for the optimization of the helical gear set, the optimized value of module and the pinion teeth, chosen from the acceptable design, are rounded off to the standard values. From the acceptable design space the feasible design solutions can be obtained. Any point lying in the acceptable design space results into an acceptable design solution of the gear, but for the optimal compact design, the point chosen should be such that it gives the minimum value of objective function i.e., volume of the pinion. Thus, out of all possible set of points lying in the acceptable design space, that point is chosen which gives minimum value of feasible volume for the stated input parameters and a given material.

When the value of module and the number of teeth on the pinion are used in the design, their values are rounded off to the standard values so as to reduce the inventory of tooling and facilitate ease and economy of manufacturing. In doing so the value of feasible volume is also slightly changed. By moving the constant objective function (volume) curve away from origin, the point of tangency between design space and the objective function curve is located. Taking the coordinates of this point of tangency of the objective function curve with constraint equation curves, a compact gear set can be designed by using the value of the rounded number of pinion teeth and the rounded standard module value. From graphs in reference [A. Kader et, al.2003], it is found that the objective function curves are always tangent to the design space curve at the point of intersection with the bending and contact stresses curves. This means, coordinates of point of intersection give optimum values of module and the number of pinion teeth, thus optimum volume can be calculated.

3. RESULTS AND DISCUSSION

For analysing the effect magnitude of different input parameters such as power, speed, gear ratio of the gear system, pressure angle, helix angle and material properties on the optimal values of the gear size, the following ranges of input parameters was taken: power (100-1000 kW), speed (500-2500 rpm), gear ratio (2-6), helix angle (15^o – 30^o), pressure angle (20^o and 25^o) and twenty different gear materials are taken. To illustrate the problem and show as an example, the effects of different parameters on the feasible optimal volume of a helical gear, a structural steel is selected from the twenty different materials. Material properties are: modulus of elasticity = 205 x 10^6 N/mm^2, permissible bending stress = 86.802 N/mm^2, permissible Hertzian stress = 745 N/mm^2, Brinell hardness = 340 and poisson's ratio = 0.3.

3.1 Effect of input Power

Fig. 1 shows the effect of input power on the feasible optimum volume of a helical gear. It is obvious that, with the increase in the power to be transmitted, the value of the volume increases thereby increasing the size of the gear set. It was observed that the relations between power and feasible optimum volume are linear in behavior. This increase in volume results in higher value of module, therefore wear of the teeth is more likely to occur, and bending mode of failure is decreased. This is also obvious from Lewis equation [Mahadevan, K., et. al., 2003] where the bending stress is inversely proportional to module.

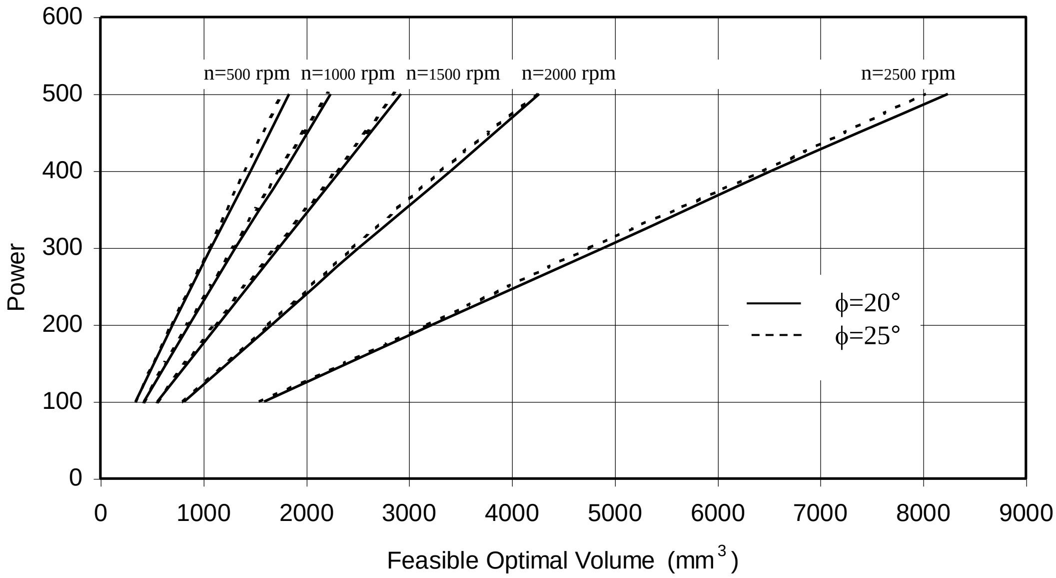

Fig. 1.: Effect of input power on feasible optimal helical gear volume

3.2 Effect of input Speed

Fig. 2 shows the effect of input speed on the feasible optimal volume of a helical gear. It is observed that there is a sudden increase in optimum volume when the speed is decreased. It is also observed that for a given decrease in speed the increase in the value of optimum volume is much larger for weaker materials similar to spur gears [A. Kader, et. al.1998].

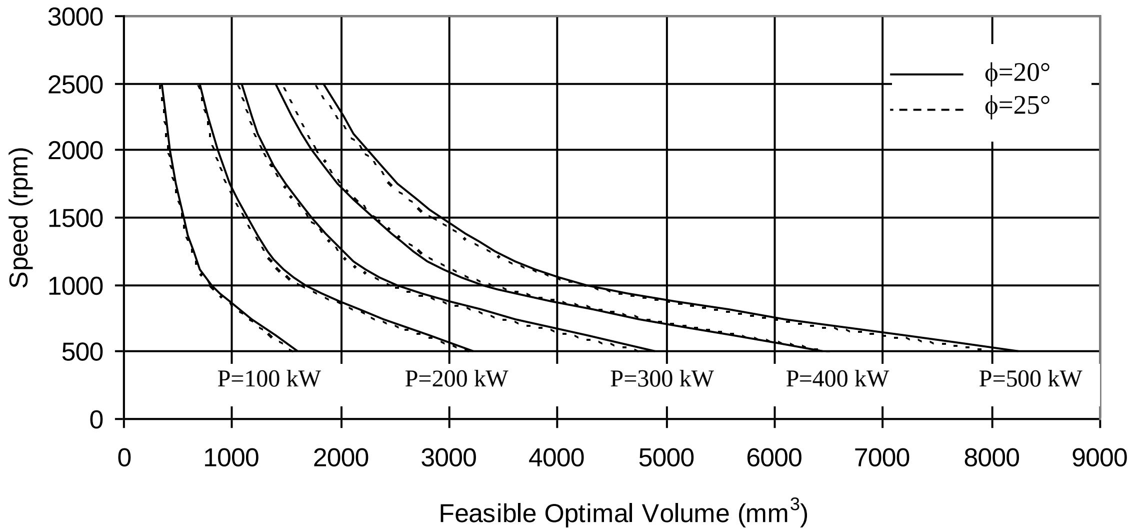

Fig. 2: Effect of input speed on feasible optimal helical gear volume

3.3. Effect of input Gear Ratio

Fig. 3 shows the effect of gear ratio on the feasible optimal volume of a helical gear. It was found that there is a slight decrease in optimal volume when the gear ratio is increased.

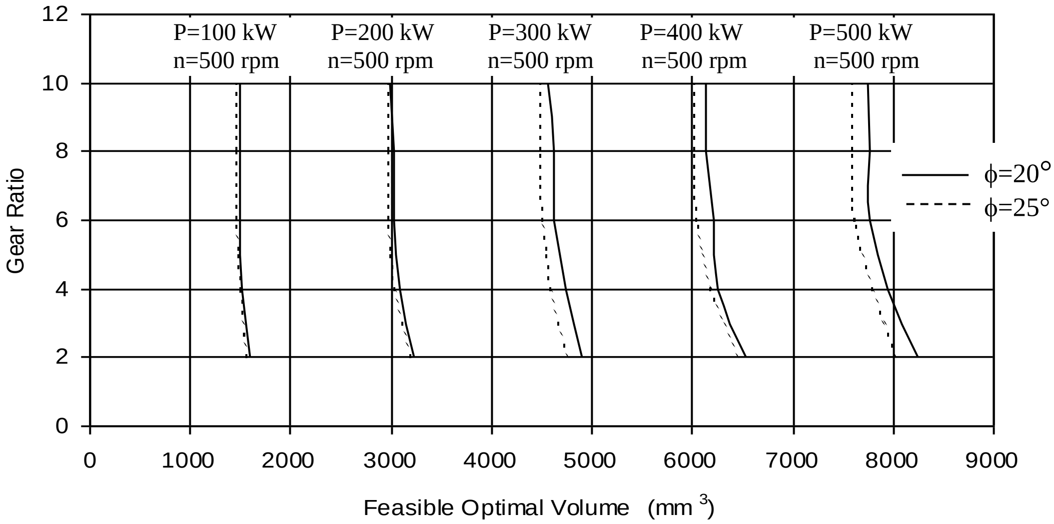

Fig. 3.: Effect of input gear ratio on feasible optimal helical gear volume

3.4 Effect of Pressure Angle and Helix Angle

From Figs. (1, 2 and 3) it can be observed that the effect of pressure angle on the feasible optimal volume is not much. The decrease in volume is not of that importance in helical gear as in spur gear, especially at lower power rather than higher power. It was also observed that the optimal volume values for helical gears are mostly occurred at higher helix angle, unlike the case of spur gears. Hence in spur gear the helix angle is equal to zero, therefore the minimum volumes are always laid at maximum permissible face width of the gear, ie. at (13 * m) [Nigam, et. al. 1992].

4. CONCLUSIONS

The following conclusions for the optimal design of compact helical gear can be drawn with the help of the above discussion:

In the conventional design process, the designer is much concerned about failure against wear and dynamic loads for a size obtained by using Lewis equation. Gear set size and margin of over safety are not of main interest for the designer in the conventional design process. However, the concept of design space givens a better insight into the design of optimized gear set. It locates the design at just the right safety limit, at the same time keeps a careful view on the size of the gear set.

a) In the conventional mode of gear design the gears are designed for bending then checked for wear. But in the design space concept, the gear is designed fulfilling the bending and wear criteria simultaneously.

b) The input power is directly proportional to the gear optimal volume size whereas the speed is inversely proportional to the gear optimal volume. Therefore the designer with help of Figs. (1 and 2), can compromise the best region to select the required power and speed keeping in mind the size of the gear set.

c) Selection of large pressure angle and helix angle together leads to a significant minimization of the optimal volume of the gear set.

d) Finally the mode of failure, which is based on the theoretical module at which failure occurs corresponding to the optimum conditions, is strongly dependent on the material properties. The stronger the material the smaller the optimal volume.

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