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On q-Analogue of Two-index and Two-Variable Generalized Hermite Polynomials

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Abstract

In this paper, we introduce the q-analogue of two-index and two-variable generalized Hermite polynomials. The recurrence relations and Rodrigue's formula for this polynomials are derived.

Keywords: q-analogue of two-index and two-variable generalized Hermite polynomials, recurrence relations, Rodrigue's formula.

1. Introduction

In this section, we will give a summary of the mathematical notations and definitions required in this paper for the convenience of the reader.

For $0 < q < 1$, $\quad [n]_q = \dfrac{1-q^n}{1-q}, \quad [n]_1 = [n],$

(1.1)

$$[n]_q! = [n]_q [n-1]_q \cdots [1]_q, \qquad n \in N \cup \{0\}, \quad [0]_q! = 1.$$

For $0 \le k \le n$ and zero otherwise the binomial coefficient is defined by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q! [k]_q!}, \qquad (1.2)$$

Or
$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q;q)_n}{(q;q)_{n-k}(q;q)_k},$$

where $(q;q)_n$ is the q-shifted factorial defined as

$$(q;q)_n = \prod_{k=0}^{n-1}\left(1 - aq^k\right), \quad n \in N \text{ with } (a;q)_0 = 1$$

The q-exponential function $e_q(x)$ is defined by

$$e_q(x) = \sum_{n=0}^{\infty} \frac{x^n}{[n]_q!} = \sum_{n=0}^{\infty} \frac{x^n}{(q;q)_n}, \qquad e_1(x) = e(x), \qquad (1.3)$$

and this function $e_q(x)$ has the following properties:

$$\frac{1}{e_q(x)} = e_{-q}(-x); \qquad (1.4)$$

$$e_q(q\ x) = (1 + (q\ -1)x)\, e_q(x). \qquad (1.5)$$

The q-derivative with index α is defined by [7]

$$D_\alpha f(x) = \frac{f(q^\alpha x) - f(x)}{(q^\alpha - 1)x}, \qquad D_1 = D \qquad (1.6)$$

which for q-derivative of the pair of functions is valid:

$$D(\lambda\, a(x) + \mu\, b(x)) = \lambda\, Da(x) + \mu\, Db(x), \qquad (1.7)$$

$$D(a(x).b(x)) = a(qx) D\, b(x) + D\, a(x) b(x), \qquad (1.8)$$

$$D\left(\frac{a(x)}{b(x)}\right) = \frac{Da(x)b(x) - a(x)Db(x)}{b(x)b(qx)}. \qquad (1.9)$$

Also, the q-analogue of $(x \pm y)^n$ is given by [6]

$$(x \pm y)^n = (x \pm y)_n = x^n \left(\mp {}^{y}\!/_{x}; q\right)_n = x^n \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{k(k-1)/2} \left(\pm {}^{y}\!/_{x}\right)^k. \quad (1.10)$$

2. q-analogue of two-index and two-variable generalized Hermite polynomials

Shrivastava [8] introduced and studied the generalization form of Hermite polynomials $H_{n,m}(x,y)$ which defined as

$$\exp[2x(t+h) - (y+1)(t+h)^2] = \sum_{n,m=0}^{\infty} H_{n,m}(x,y) \frac{t^n h^m}{n!m!}.$$

Here, we introduce the generalized q-analogue form of the above polynomials, denoted by $H_{n,m}^{(k)}(x,y;q)$.

Let $0 < q < 1$, for $n, m = 0,1,2,\ldots$ and k a positive integer, we defined the q-analogue generalized Hermite polynomials of two-index and two-variable by:

$$F(x,y,t,h;q) = \frac{\exp_q(2x(t+h))}{\exp_q(qy(t+h)^k)} = \sum_{n,m=0}^{\infty} H_{n,m}^{(k)}(x,y;q) t^n h^m. \qquad (2.1)$$

Now, expanding the generating function (2.1) in series, we get

$$\sum_{n,m=0}^{\infty} H_{n,m}^{(k)}(x,y;q)t^n h^m = \exp_q(2x(t+h))\exp_{-q}\left(-qy(t+h)^k\right)$$

$$= \sum_{n=0}^{\infty} \frac{(2x)^n (t+h)^n}{[n]_q!} \sum_{r=0}^{\infty} \frac{(-qy)^r (t+h)^{kr}}{[r]_{-q}!}$$

by using relation (1.10), we obtain

$$\sum_{n,m=0}^{\infty} H_{n,m}^{(k)}(x,y;q)t^n h^m = \sum_{n=0}^{\infty} \frac{(2x)^n}{[n]_q!} \sum_{m=0}^{\infty} \begin{bmatrix} n \\ m \end{bmatrix}_q q^{\binom{m}{2}} t^{n-m} h^m \sum_{r=0}^{\infty} \frac{(-qy)^r}{[r]_{-q}!} \sum_{s=0}^{\infty} \begin{bmatrix} kr \\ s \end{bmatrix}_{-q} q^{\binom{s}{2}} t^{kr-s} h^s$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{n} q^{\binom{m}{2}} \frac{(2x)^n t^{n-m} h^m}{[n-m]_q![m]_q!} \sum_{r=0}^{\infty} \sum_{s=0}^{kr} q^{\binom{s}{2}} \frac{(-1)^r [kr]_{-q}!(qy)^r t^{kr-s} h^s}{[r]_{-q}![kr-s]_{-q}![s]_{-q}!}$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} q^{\binom{m}{2}} \frac{(2x)^{n+m} t^n h^m}{[n]_q![m]_q!} \sum_{r=0}^{\infty} \sum_{s=0}^{r} q^{\binom{ks}{2}} \frac{(-1)^r [kr]_{-q}!(qy)^r t^{kr-ks} h^{ks}}{[r]_{-q}![kr-ks]_{-q}![ks]_{-q}!}$$

$$= \sum_{n,m=0}^{\infty} \sum_{r=0}^{[n/k]} \sum_{s=0}^{[m/k]} q^{\binom{m}{2}+\binom{ks}{2}} \frac{(-1)^{r+s} [kr+ks]_{-q}!(2x)^{n+m-kr-ks}(qy)^{r+s} t^n h^m}{[n-kr]_q![m-ks]_q![r+s]_{-q}![kr]_{-q}![ks]_{-q}!}$$

Equating the coefficients of $t^n h^m$ in both sides, we obtain an explicit representation for $H_{n,m}^{(k)}(x,y;q)$ in the form

$$H_{n,m}^{(k)}(x,y;q) = \sum_{r=0}^{[n/k]} \sum_{s=0}^{[m/k]} q^{\binom{m}{2}+\binom{ks}{2}} \frac{(-1)^{r+s} [kr+ks]_{-q}!(2x)^{n+m-kr-ks}(qy)^{r+s}}{[n-kr]_q![m-ks]_q![r+s]_{-q}![kr]_{-q}![ks]_{-q}!}. \qquad (2.2)$$

3. Recurrence relations and Rodrigue's formula

Theorem 3.1

The q-analogueof two-index and two-variable generalized Hermite polynomials $H_{n,m}^{(k)}(x,y;q)$ satisfy the following relations:

$$\frac{\partial^s}{\partial x^s} H_{n,m}^{(k)}(x,y;q) = 2^s \sum_{r=0}^{[s]} q^{\binom{r}{2}} \frac{[s]_q! H_{n+r-s,m-r}^{(k)}(x,y;q)}{[s-r]_q![r]_q!}, \qquad (3.1)$$

and

$$\frac{\partial^s}{\partial y^s} H_{n,m}^{(k)}(x,y;q) = (-1)^s q^s \sum_{r=0}^{[sk]} q^{\binom{r}{2}} \frac{[sk]_q! H_{n+r-sk,m-r}^{(k)}(x,y;q)}{[sk-r]_q![r]_q!}. \qquad (3.2)$$

Proof.

Differentiating (2.1) with respect to x yields

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}H_{n,m}^{(k)}(x,y;q)t^n h^m = 2(t+h)\frac{\exp_q(2x(t+h))}{\exp_q\left(qy(t+h)^k\right)}$$

$$= 2\sum_{r=0}^{1}\frac{t^{1-r}h^r}{[1-r]_q![r]_q!}\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^n h^m$$

$$= 2\sum_{n,m=0}^{\infty}\sum_{r=0}^{1}\frac{H_{n+r-1,m-r}^{(k)}(x,y;q)}{[1-r]_q![r]_q!}t^n h^m$$

On equating the coefficients of $t^n h^m$ on both sides of the above equation, we obtain

$$\frac{\partial}{\partial x}H_{n,m}^{(k)}(x,y;q) = 2\sum_{r=0}^{1}q^{\binom{r}{2}}\frac{H_{n+r-1,m-r}^{(k)}(x,y;q)}{[1-r]_q![r]_q!}, \tag{3.3}$$

thus

$$\frac{\partial^2}{\partial x^2}H_{n,m}^{(k)}(x,y;q) = 2^2\sum_{r=0}^{[2]}q^{\binom{r}{2}}\frac{[2]_q!H_{n+r-2,m-r}^{(k)}(x,y;q)}{[2-r]_q![r]_q!}, \tag{3.4}$$

by continuing this process, we get the required relation (3.1).
Again, differentiating (2.1) with respect to y yields

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial y}H_{n,m}^{(k)}(x,y;q)t^n h^m = -q(t+h)^k\frac{\exp_q(2x(t+h))}{\exp_q\left(qy(t+h)^k\right)}$$

$$= -q\sum_{r=0}^{[k]}q^{\binom{r}{2}}\frac{[k]_q!t^{k-r}h^r}{[k-r]_q![r]_q!}\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^n h^m$$

$$= -q\sum_{n,m=0}^{\infty}\sum_{r=0}^{[k]}q^{\binom{r}{2}}\frac{[k]_q!H_{n+r-k,m-r}^{(k)}(x,y;q)}{[k-r]_q![r]_q!}t^n h^m$$

On equating the coefficients of $t^n h^m$ on both sides of the above equation, we get

$$\frac{\partial}{\partial y}H_{n,m}^{(k)}(x,y;q) = -q\sum_{r=0}^{[k]}q^{\binom{r}{2}}\frac{[k]_q!H_{n+r-k,m-r}^{(k)}(x,y;q)}{[k-r]_q![r]_q!}, \tag{3.5}$$

thus

$$\frac{\partial^2}{\partial y^2} H_{n,m}^{(k)}(x,y;q) = q^2 \sum_{r=0}^{[2k]} q^{\binom{r}{2}} \frac{[2k]_q! H_{n+r-2k,m-r}^{(k)}(x,y;q)}{[2k-r]_q! [r]_q!}, \qquad (3.6)$$

by continuing this process, we get the required result (3.2).

Theorem 3.2

The polynomials sequence $\{H_{n,m}^{(k)}(x,y;q)\}_{n,m=0}^{\infty}$ satisfies the next recurrence relation

$$[n+1]_q H_{n+1,m}^{(k)}(x,y;q) - (1-q)qy \sum_{r=0}^{[k]} q^{\binom{r}{2}} \frac{[n-k+r+1]_q [k]_q! H_{n-k+r+1,m-r}^{(k)}(x,y;q)}{[k-r]_q! [r]_q!}$$
$$= 2x H_{n,m}^{(k)}(x,y;q) - kqy \sum_{r=0}^{[k-1]} q^{\binom{r}{2}} \frac{[k-1]_q! H_{n-k+r+1,m-r}^{(k)}(x,y;q)}{[k-r-1]_q! [r]_q!}$$

, (3.7)

and

$$[m+1]_q H_{n,m+1}^{(k)}(x,y;q) - (1-q)qy \sum_{r=0}^{[k]} q^{\binom{r}{2}} \frac{[m-r+1]_q [k]_q! H_{n-k+r,m-r}^{(k)}(x,y;q)}{[k-r]_q! [r]_q!}$$
$$= 2x H_{n,m}^{(k)}(x,y;q) - kqy \sum_{r=0}^{[k-1]} q^{\binom{r}{2}} \frac{[k-1]_q! H_{n-k+r,m-r}^{(k)}(x,y;q)}{[k-r-1]_q! [r]_q!}$$

. (3.8)

Proof.

Differentiating (2.1) with respect to t and using (1.9), we find

$$\sum_{n,m=0}^{\infty} \frac{\partial}{\partial t} H_{n,m}^{(k)}(x,y;q) t^n h^m$$
$$= \frac{2x \exp_q(2x(t+h)) \exp_q\left(qy(t+h)^k\right) - kqy(t+h)^{k-1} \exp_q(2x(t+h)) \exp_q\left(qy(t+h)^k\right)}{\exp_q\left(qy(t+h)^k\right) \exp_q\left(qy(qt+h)^k\right)}$$

$$\sum_{n,m=0}^{\infty} [n]_q H_{n,m}^{(k)}(x,y;q) t^{n-1} h^m = \frac{\left(2x - kqy(t+h)^{k-1}\right) \exp_q(2x(t+h))}{\exp_q\left(qy(qt+h)^k\right)}$$

By using the relation (1.5), we get

$$\sum_{n,m=0}^{\infty} [n]_q H_{n,m}^{(k)}(x,y;q) t^{n-1} h^m = \frac{\left(2x - kqy(t+h)^{k-1}\right) \exp_q(2x(t+h))}{\left(1-(1-q)qy(t+h)^k\right) \exp_q\left(qy(t+h)^k\right)}$$

$$\sum_{n,m=0}^{\infty}[n]_q H_{n,m}^{(k)}(x,y;q)t^{n-1}h^m-(1-q)qy\sum_{r=0}^{k}q^{\binom{r}{2}}\frac{[k]_q!t^{k-r}h^r}{[k-r]_q![r]_q!}\sum_{n,m=0}^{\infty}[n]_q H_{n,m}^{(k)}(x,y;q)t^{n-1}h^m$$

$$=2x\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^nh^m-kqy\sum_{r=0}^{k-1}q^{\binom{r}{2}}\frac{[k-1]_q!t^{k-r-1}h^r}{[k-r-1]_q![r]_q!}\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^nh^m$$

$$\sum_{n,m=0}^{\infty}[n+1]_q H_{n+1,m}^{(k)}(x,y;q)t^nh^m-(1-q)qy\sum_{n,m=0}^{\infty}\sum_{r=0}^{k}q^{\binom{r}{2}}\frac{[n-k+r+1]_q[k]_q!H_{n-k+r+1,m-r}^{(k)}(x,y;q)}{[k-r]_q![r]_q!}t^nh^m$$

$$=2x\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^nh^m-kqy\sum_{n,m=0}^{\infty}\sum_{r=0}^{k-1}q^{\binom{r}{2}}\frac{(q;q)_{k-1}H_{n-k+r+1,m-r}^{(k)}(x,y;q)}{[k-r-1]_q![r]_q!}t^nh^m$$

Now, on equating the coefficients of t^nh^m of the above equation, we get the required relation (3.7).

Again, differentiating (2.1) with respect to h, we find

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial h}H_{n,m}^{(k)}(x,y;q)t^nh^m$$

$$=\frac{2x\exp_q(2x(t+h))\exp_q\left(qy(t+h)^k\right)-kqy(t+h)^{k-1}\exp_q(2x(t+h))\exp_q\left(qy(t+h)^k\right)}{\exp_q\left(qy(t+h)^k\right)\exp_q\left(qy(qt+h)^k\right)}$$

$$\sum_{n,m=0}^{\infty}[m]_q H_{n,m}^{(k)}(x,y;q)t^nh^{m-1}=\frac{\left(2x-kqy(t+h)^{k-1}\right)\exp_q(2x(t+h))}{\left(1-(1-q)qy(t+h)^k\right)\exp_q\left(qy(t+h)^k\right)}$$

$$\sum_{n,m=0}^{\infty}[m+1]_q H_{n,m+1}^{(k)}(x,y;q)t^nh^m-(1-q)qy\sum_{n,m=0}^{\infty}\sum_{r=0}^{k}q^{\binom{r}{2}}\frac{[m-r+1]_q[k]_q!H_{n-k+r,m-r}^{(k)}(x,y;q)}{[k-r]_q![r]_q!}t^nh^m$$

$$=2x\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^nh^m-kqy\sum_{n,m=0}^{\infty}\sum_{r=0}^{k-1}q^{\binom{r}{2}}\frac{[k-1]_q!H_{n-k+r,m-r}^{(k)}(x,y;q)}{[k-r-1]_q![r]_q!}t^nh^m$$

Now, on comparing of coefficients of t^nh^m of the above equation, we get the required relation (3.8).

Theorem 3.3

For the polynomial $H_{n,m}^{(k)}(x,y;q)$ the following result holds true:

$$q^{\binom{m}{2}}\frac{(2x)^{n+m}}{[n]_q![m]_q!}=\sum_{r=0}^{[n/k]}\sum_{s=0}^{[m/k]}q^{\binom{ks}{2}}\frac{[kr+ks]_q!(qy)^{r+s}}{[r+s]_q![kr]_q![ks]_q!}H_{n-kr,m-ks}^{(k)}(x,y;q). \quad (3.9)$$

Proof. Using generating function (2.1) and relation (1.3), we have

$$\exp_q\left(2x(t+h)\right)=\exp_q\left(qy(t+h)^k\right)\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^n h^m$$

$$\sum_{n=0}^{\infty}\frac{(2x)^n(t+h)^n}{[n]_q!}=\sum_{r=0}^{\infty}\frac{(qy)^r(t+h)^{kr}}{[r]_q!}\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^n h^m,$$

by using the relation (1.10), we find

$$\sum_{n=0}^{\infty}\sum_{m=0}^{[n]}q^{\binom{m}{2}}\frac{(2x)^n t^{n-m}h^m}{[n-m]_q![m]_q!}=\sum_{r=0}^{\infty}\sum_{s=0}^{[kr]}q^{\binom{s}{2}}\frac{[kr]_q!(qy)^r t^{kr-s}h^s}{[r]_q![kr-s]_q![s]_q!}\sum_{n,m=0}^{\infty}H_{n,m}^{(k)}(x,y;q)t^n h^m$$

$$\sum_{n,m=0}^{\infty}q^{\binom{m}{2}}\frac{(2x)^{n+m}t^n h^m}{[n]_q![m]_q!}=\sum_{n,m=0}^{\infty}\sum_{r,s=0}^{\infty}q^{\binom{ks}{2}}\frac{[kr+ks]_q!(qy)^{r+s}}{[r+s]_q![kr]_q![ks]_q!}H_{n,m}^{(k)}(x,y;q)t^{n+kr}h^{m+ks}$$

$$=\sum_{n,m=0}^{\infty}\sum_{r=0}^{[n/k]}\sum_{s=0}^{[m/k]}q^{\binom{ks}{2}}\frac{[kr+ks]_q!(qy)^{r+s}}{[r+s]_q![kr]_q![ks]_q!}H_{n-kr,m-ks}^{(k)}(x,y;q)t^n h^m,$$

by equating the coefficients of $t^n h^m$ of the above equation, we obtain the required relation (3.9) .

Theorem 3.4

Let $q\in(0,1)$, the q-analogue generalized Hermite polynomials $H_{n,m}^{(k)}(x,y;q)$ has the following Rodrigue's formula:

$$H_{n,m}^{(k)}(x,y;q)=\exp\left(-2^{-k}qy\frac{d^k}{dx^k}\right)q^{\binom{m}{2}}\frac{(2x)^{n+m}}{[n]_q![m]_q!}\quad .$$

(3.10)

Proof. Since,

$$\frac{\partial}{\partial x}\exp_q\left(2x(t+h)\right)=2(t+h)\exp_q\left(2x(t+h)\right)$$

and

$$\frac{\partial^2}{\partial x^2}\exp_q\left(2x(t+h)\right)=2^2(t+h)^2\exp_q\left(2x(t+h)\right),$$

by continuing this process, we get

$$\frac{\partial^k}{\partial x^k}\exp_q\left(2x(t+h)\right)=2^k(t+h)^k\exp_q\left(2x(t+h)\right)$$

$$\therefore \left[2^{-1}\frac{\partial}{\partial x}\right]^{k} \exp_q\left(2x(t+h)\right) = (t+h)^k \exp_q\left(2x(t+h)\right). \qquad (3.11)$$

Thus,

$$\exp_q\left[-qy2^{-k}\frac{d^k}{dx^k}\right]\exp_q\left(2x(t+h)\right) = \sum_{m=0}^{\infty}\frac{(-1)^m(qy)^m}{[m]_q!}\left[2^{-1}\frac{d}{dx}\right]^{mk}\exp_q\left(2x(t+h)\right)$$

$$= \sum_{m=0}^{\infty}\frac{(-1)^m(qy)^m}{[m]_q!}(t+h)^{mk}\exp_q\left(2x(t+h)\right)$$

$$= \exp_q\left(2x(t+h) - qy(t+h)^k\right)$$

$$= \sum_{n,m=0}^{\infty} H_{n,m}^{(k)}(x,y;q)t^n h^m \qquad (3.12)$$

Hence,

$$\sum_{n,m=0}^{\infty} H_{n,m}^{(k)}(x,y;q)t^n h^m = \exp_q\left[-qy2^{-k}\frac{d^k}{dx^k}\right]\sum_{n=0}^{\infty}\frac{(2x)^n(t+h)^n}{[n]_q!}$$

$$= \exp_q\left[-qy2^{-k}\frac{d^k}{dx^k}\right]\sum_{n=0}^{\infty}\sum_{m=0}^{[n]} q^{\binom{m}{2}}\frac{(2x)^n t^{n-m}h^m}{[n-m]_q![m]_q!}$$

$$= \exp_q\left[-qy2^{-k}\frac{d^k}{dx^k}\right]\sum_{n,m=0}^{\infty} q^{\binom{m}{2}}\frac{(2x)^{n+m} t^n h^m}{[n]_q![m]_q!}$$

By equating the coefficientsof $t^n h^m$, we find the required relation (3.10).

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كثيرات حدود هرميت ذات دليلين ومتغيرين المعممة من النوع كيو

فضل بن فضل محسن و فضل صالح ناصر السرحي
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الملخص
في هذه الورقة قدمنا كثيرات حدودهرميت ذات دليلين ومتغيرين المعممة من النوع كيو وأيضاً أثبتنا العلاقات التكرارية و صيغة رودريجس لها.
الكلمات المفتاحية : كثيرات حدود هرميت ذات دليلين ومتغيرين المعممة من النوع كيو, العلاقات التكرارية , صيغة رودريجس.